Key

Math 4

Name________________________

3-5 Adding and Subtracting Rational Expressions

Date________

Learning goal:
I can manipulate rational expressions into useful equivalent forms by simplifying, adding, subtracting, multiplying, and dividing.

Add and simplify the following fractions.

1) $\frac{4}{5}+\frac{4}{7} = \frac{28}{35}+\frac{20}{35} = \boxed{\frac{48}{35}}$

2) $\frac{5}{4}+\frac{3}{4} = \boxed{\frac{8}{4}}$

3) $\frac{1}{2}+\frac{11}{8} = \frac{4}{8}+\frac{11}{8} = \boxed{\frac{15}{8}}$

4) $\frac{5}{4}+\frac{4}{3} = \frac{15}{12}+\frac{16}{12} = \boxed{\frac{31}{12}}$

5) $\frac{1}{5}+\frac{1}{3} = \frac{3}{15}+\frac{5}{15} = \boxed{\frac{8}{15}}$

6) $\frac{8}{7}+\frac{4}{3} = \frac{24}{21}+\frac{28}{21} = \boxed{\frac{52}{21}}$

Find each difference.

7) $\frac{5}{3}-\frac{2}{5} = \frac{25}{15}-\frac{6}{15} = \boxed{\frac{19}{15}}$

8) $\frac{7}{4}-\frac{6}{7} = \frac{49}{28}-\frac{24}{28} = \boxed{\frac{25}{28}}$

9) $\frac{7}{6}-\frac{1}{3} = \frac{7}{6}-\frac{2}{6} = \boxed{\frac{5}{6}}$

10) $\frac{4}{3}-\frac{6}{7} = \frac{28}{21}-\frac{18}{21} = \boxed{\frac{10}{21}}$

11) $\frac{5}{3}-\frac{1}{2} = \frac{10}{6}-\frac{3}{6} = \boxed{\frac{7}{6}}$

12) $\frac{2}{1}-\frac{1}{6} = \frac{12}{6}-\frac{1}{6} = \boxed{\frac{11}{6}}$

Adding and Subtracting Rational Expressions

As a review, adding and subtracting *fractions* requires the fractions to have the same denominator. If they already have the same denominator, combine the numerators over that denominator:

$$\frac{9}{8} - \frac{6}{8} = \frac{9-6}{8} = \frac{3}{8}$$

If they have different denominators, you have to create a common denominator, then combine the numerators:

$$\frac{2}{5} + \frac{1}{2} = \frac{2(2)}{5(2)} + \frac{1(5)}{2(5)} = \frac{4}{10} + \frac{5}{10} = \frac{4+5}{10} = \frac{9}{10}$$

And, once you complete the operation, always check to see if you can simplify the result:

$$\frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1 \cdot \cancel{3}}{2 \cdot \cancel{3}} = \frac{1}{2}$$

Adding and subtracting **rational expressions** works exactly the same way – you just have some variables involved and, perhaps, more complex factors. If the denominators look exactly the same, *no matter how complex they are*, then you can combine the numerators over that denominator. If the denominators are different, then you have to find a common denominator and modify the expressions before doing any operations with the numerators.

Same denominators: Combine numerators and simplify, if possible

1) $\frac{5}{x} + \frac{3+x}{x} = \frac{5+3+x}{x} = \frac{8+x}{x}$

2) $\frac{2x+5}{x^2-4} - \frac{x+3}{x^2-4} = \frac{(2x+5)-(x+3)}{x^2-4} = \frac{2x+5-x-3}{x^2-4} = \frac{2x-x+5-3}{x^2-4}$

$$= \frac{x+2}{x^2-4}$$

$$= \frac{\cancel{x+2}}{\cancel{(x+2)}(x-2)}$$

$$= \frac{1}{x-2}$$

Same denominators ***except for sign***: Remember in 6.1 how we changed opposite factors into the same factor by factoring out a negative 1? If you have denominators that are the same except for opposite signs, you can make them the same denominator by doing the same thing. The -1 that is factored out can be moved around, like to the numerator.

$$\text{Remember: } \frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$$

3) $$\frac{5x}{y-4} + \frac{2x}{4-y} = \frac{5x}{y-4} + \frac{2x}{-(-4+y)} = \frac{5x}{y-4} + \frac{-2x}{y-4} = \frac{5x-2x}{y-4} = \frac{3x}{y-4}$$

factor out a -1 — move the negative sign to the numerator — switch the order — sign moved

Different denominators: Just like with fractions, if the denominators of our rational expressions are different, we have to make them the same in order to add or subtract the expressions – and we want a <u>least</u> common denominator (LCD) (that is, with the fewest number of factors).

A common denominator contains within it <u>all</u> the factors of <u>each</u> denominator.

Rule 1. If you're looking at simple products of variables, like x^5y^2 and x^3y^4, then use the <u>highest exponent</u> of each variable in the common denominator:

$$x^5y^2 \quad x^3y^4 \quad \rightarrow \quad x^5y^4 = \text{LCD}$$

Rule 2. If you have numerical coefficients as well as variables, like $6x$, $12xy$, and $24x^2$, factor out any number common to all the coefficients first, then see what's left. Use this common factor only once! Then, see if you have a factor common to the remaining terms – use this factor only once (continue as needed). Then, include any other factors left over from the different denominators. Remember, the common denominator must include <u>all</u> factors of <u>each</u> denominator <u>within it</u>.

$6x$	$12xy$	$24x^2$
$6 \cdot x$	$6 \cdot 2 \cdot x \cdot y$	$6 \cdot 2 \cdot 2 \cdot x^2$

common factor (to all three): 6; common to two: 2; additional factor: 2

$$6 \cdot 2 \cdot 2 \cdot x^2 \cdot y = 24x^2y = \text{LCD}$$

highest powers of variables: $x^2 \cdot y$

Rule 3. If you're dealing with polynomials, like $x^2 + 4x - 12x$, you must first factor the polynomials of each denominator (if possible). Then use those factors to build a common denominator.

$$x^2 + 4x - 12 \qquad\qquad x^2 + 2x - 8$$
$$(x - 2)(x + 6) \qquad\qquad (x + 4)(x - 2)$$

common factor

$$\mathbf{(x - 2)(x + 6)(x + 4) \quad = \text{LCD}}$$

After you find the LCD, you'll have to multiply the numerator and denominator of each rational expression by whatever factors are missing *compared to the common denominator.*

For example, to convert $\dfrac{1}{12xy}$ to an expression with an **LCD** of $24x^2y$ ($= 12xy \cdot 2x$), multiply top and bottom by $2x$:

$$\frac{1}{12xy} \cdot \frac{2x}{2x} = \frac{2x}{24x^2y}$$

To convert $\dfrac{x+3}{(x+6)(x-2)}$ to an expression with an **LCD** of $(x + 6)(x + 4)(x - 2)$, multiply top and bottom by the "missing factor" $(x + 4)$:
$$\frac{x+3}{(x+6)(x-2)} \cdot \frac{(x+4)}{(x+4)} = \frac{(x+3)(x+4)}{(x+6)(x-2)(x+4)}.$$

Do not be tempted to simplify these expressions again! You're creating them so you can complete the other operations of addition and subtraction with the numerators.

Example 1. Perform the indicated operation and simplify: $\dfrac{3y}{x+2} + \dfrac{y-1}{x+2}$

Step 1. Do the fractions have the same denominator? Yes!

Step 2. Add the numerators over the common denominator.
$$\frac{3y}{x+2} + \frac{y-1}{x+2} = \frac{3y+y-1}{x+2}$$
$$= \frac{4y-1}{x+2}$$

Step 3. Can the resulting expression be simplified? No. **Done!**

Example 2. Perform the indicated operation and simplify: $\frac{x^2}{x+3} - \frac{2x+15}{x+3}$

Step 1. Do the fractions have the same denominator? Yes!

Step 2. Add the numerators over the common denominator.

$$\frac{x^2}{x+3} - \frac{2x+15}{x+3} = \frac{x^2-(2x+15)}{x+3}$$

Don't forget to distribute that minus sign!

$$= \frac{x^2-2x-15}{x+3}$$

Step 3. Can the resulting expression be simplified? Well,

$$\frac{x^2-2x-15}{x+3} = \frac{(x-5)(x+3)}{(x+3)}$$

let's try factoring the numerator and see what we get.

$$\frac{(x-5)\cancel{(x+3)}}{\cancel{(x+3)}} = x-5$$

Done!

Example 3. Perform the indicated operation and simplify: $\frac{x+1}{9x} + \frac{2x-4}{6x}$

Step 1. Do the fractions have the same denominator? Not quite!

Step 2. Find the LCD. First factor the denominators, then include all the factors of each denominator in a common denominator

9x → $3 \cdot 3 \cdot x$ **6x** → $3 \cdot 2 \cdot x$

common factor → $3 \cdot 3 \cdot 2 \cdot x$ ← highest power of the variable

$$= \mathbf{18x} = \mathbf{LCD}$$

Step 3. Multiply top and bottom of each rational expression by whatever factor is "missing" in the denominator compared to the LCD.

$$\frac{x+1}{9x} \cdot \frac{2}{2} + \frac{2x-4}{6x} \cdot \frac{3}{3}$$

$$= \frac{2(x+1)}{18x} + \frac{3(2x-4)}{18x}$$

Step 4. Add the numerators over the common denominator.

$$= \frac{2(x+1)+3(2x-4)}{18x}$$

$$= \frac{2x+2+6x-12}{18x}$$

Combine like terms.

$$= \frac{2x+6x+2-12}{18x}$$

$$= \frac{8x-10}{18x}$$

Step 5. Can the resulting expression be simplified? $$= \frac{2(4x-5)}{2 \cdot 9x}$$

Try factoring. $$= \frac{\cancel{2}(4x-5)}{\cancel{2} \cdot 9x} = \frac{4x-5}{9x}$$

Done.

Example 4. Perform the indicated operation and simplify: $$\frac{x^2}{x^2-1} + \frac{4x}{x^2-x}$$

Step 1. Do the fractions have the same denominator? No.

Step 2. Find the LCD. First factor the denominators, then make sure all the factors of each denominator are included within a common denominator.

$$x^2-1 \qquad\qquad x^2-x$$

$$(x-1)(x+1) \qquad\qquad x(x-1)$$

common factor

$$(x-1)(x+1)x = \text{LCD}$$

Step 3. Multiply top and bottom of each rational expression by whatever factor is "missing" in the denominator compared to the LCD.

$$\frac{x^2}{(x-1)(x+1)} \cdot \frac{x}{x} + \frac{4x}{x(x-1)} \cdot \frac{(x+1)}{(x+1)}$$

$$= \frac{x^3}{x(x-1)(x+1)} + \frac{4x(x+1)}{x(x-1)(x+1)}$$

Step 4. Add the numerators over the common denominator.

$$= \frac{x^3+4x(x+1)}{x(x-1)(x+1)}$$

$$= \frac{x^3+4x^2+4x}{x(x-1)(x+1)}$$

Step 5. Can the resulting expression be simplified?

$$= \frac{x(x^2+4x+4)}{x(x-1)(x+1)}$$

Try factoring.

$$= \frac{x(x+2)(x+2)}{x(x-1)(x+1)}$$

$$= \frac{\cancel{x}(x+2)^2}{\cancel{x}(x-1)(x+1)}$$

$$= \frac{(x+2)^2}{(x-1)(x+1)}$$

Done.

Practice Problems – Show work on a separate sheet of paper!

Add, subtract and simplify if possible.

1) $\frac{1}{x}+\frac{5}{x^2}$

2) $\frac{8}{a^2b}-\frac{6}{ab^3}$

3) $\frac{a+3}{3}-\frac{6-a}{6}$

4) $\frac{2x-3}{3x^2}+\frac{3x+3}{9x}$

5) $\frac{5}{x-1}+\frac{5}{x+1}$

6) $\frac{3x}{x^2-25}-\frac{4}{5-x}$

7) $\frac{1}{x^2-3x-10}+\frac{1}{x^2+9x+14}$

8) $\frac{3x+1}{x^2+4x+3}-\frac{x+6}{x^2-3x-4}$

9) $\frac{3}{y-z}+\frac{3}{y+z}-\frac{6z}{z^2-y^2}$

10) $\frac{1}{m}-\frac{2}{m^2}+\frac{2}{m+1}$

Answers

1) $\frac{x+5}{x^2}$

2) $\frac{8b^2-6a}{a^2b^3}$

3) $\frac{a}{2}$

4) $\frac{x^2+3x-3}{3x^2}$

5) $\frac{10x}{(x-1)(x+1)}$

6) $\frac{7x+20}{(x-5)(x+5)}$

7) $\frac{2x+2}{(x+7)(x+2)(x-5)}$

8) $\frac{2x-22}{(x+3)(x-4)}$

9) $\frac{6}{y-z}$

10) $\frac{3m^2-m-2}{m^2(m+1)}$

M4 | 3-5 HW ANSWERS

1.) $\frac{x}{x^2} + \frac{5}{x^2} = \boxed{\frac{x+5}{x^2}}$

2.) $\frac{8b^2}{a^2b^3} - \frac{6a}{a^2b^3} = \boxed{\frac{8b^2-6a}{a^2b^3}}$

3.) $\frac{2(a+3)}{6} - \frac{6-a}{6} = \frac{2a+6-6+a}{6} = \frac{3a}{6} = \boxed{\frac{a}{2}}$

4.) $\frac{3(2x-3)}{9x^2} + \frac{x(3x+3)}{9x^2} = \frac{6x-9+3x^2+3x}{9x^2} = \frac{3(x^2+3x-3)}{9x^2}$

$= \boxed{\frac{x^2+3x-3}{3x^2}}$

5.) $\frac{5(x+1)}{(x-1)(x+1)} + \frac{5(x-1)}{(x-1)(x+1)} = \frac{5x+5+5x-5}{(x-1)(x+1)} = \boxed{\frac{10x}{(x-1)(x+1)}}$

6.) $\frac{3x}{(x-5)(x+5)} - \frac{4}{-(x-5)} = \frac{3x}{(x-5)(x+5)} + \frac{4(x+5)}{(x-5)(x+5)} = \frac{3x+4x+20}{(x-5)(x+5)}$

$= \boxed{\frac{7x+20}{(x-5)(x+5)}}$

7.) $\frac{1}{(x-5)(x+2)} + \frac{1}{(x+7)(x+2)} = \frac{x+7}{(x-5)(x+2)(x+7)} + \frac{x-5}{(x-5)(x+2)(x+7)}$

$= \boxed{\frac{2x+2}{(x-5)(x+2)(x+7)}}$

8.) $\frac{3x+1}{(x+3)(x+1)} - \frac{x+6}{(x-4)(x+1)} = \frac{(3x+1)(x-4)}{(x+3)(x+1)(x-4)} - \frac{(x+6)(x+3)}{(x+3)(x+1)(x-4)}$

$= \frac{(3x^2-11x-4)-(x^2+9x+18)}{(x+3)(x+1)(x-4)} = \frac{2x^2-20x-22}{} = \frac{2(x^2-10x-11)}{} = \frac{2(x-11)\cancel{(x+1)}}{(x+3)\cancel{(x+1)}(x-4)}$

$= \boxed{\frac{2(x-11)}{(x+3)(x-4)}}$

9.) $\frac{3}{y-z} + \frac{3}{y+z} - \frac{6z}{(z+y)(z-y)} = \frac{3}{y-z} + \frac{3}{y+z} + \frac{6z}{(y+z)(y-z)}$

$= \frac{3(y+z) + 3(y-z) + 6z}{(y+z)(y-z)} = \frac{3y + 3z + 3y - 3z + 6z}{(y+z)(y-z)}$

$= \frac{6y+6z}{(y+z)(y-z)} = \frac{6(y+z)}{(y+z)(y-z)} = \boxed{\frac{6}{y-z}}$

10.) $\frac{1}{m} - \frac{2}{m^2} + \frac{2}{m+1} = \frac{m(m+1) - 2(m+1) + 2m^2}{m^2(m+1)}$

$= \frac{m^2+m-2m-2+2m^2}{m^2(m+1)} = \frac{3m^2-m-2}{m^2(m+1)} = \boxed{\frac{(3m+2)(m-1)}{m^2(m+1)}}$